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ON STABILITY ANALYSIS WITH
DISEQUILIBRIUM AWARENESS

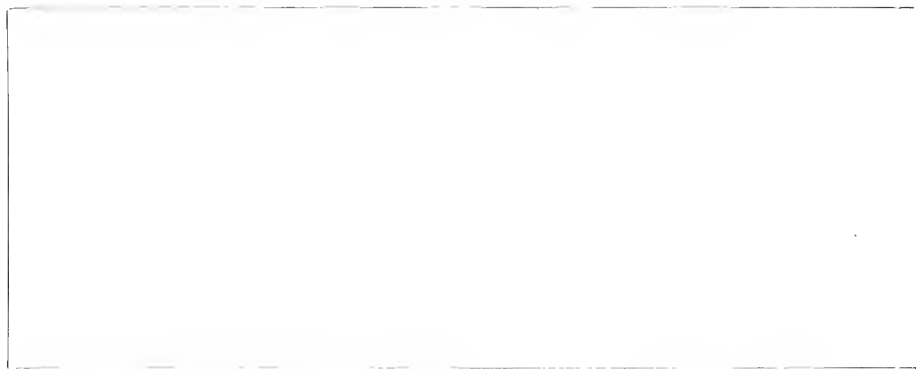
Dale O. Stahl, II
and
Franklin M. Fisher

No. 432

October 1986

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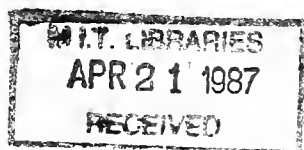
ABSTRACT

Stahl, Dale O., and Fisher, Franklin, M. -- On Stability Analysis With Disequilibrium Awareness

Most models of general equilibrium stability require agents stupidly to believe that prices are Walrasian and transactions will be completed. This paper considers models where agents, while naive about prices, are fully informed about transaction difficulties. Agents fully understand the workings and outcome of a deterministic trading mechanism mapping expressed demands into actual trades. They have rational expectations (perfect foresight) about the outcome of that mechanism. Prices respond as long as actual trades differ from notional demands. The result is shown to be an Edgeworth process and hence globally stable, converging to a Walrasian equilibrium. J. Econ. Theory,

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Journal of Economic Literature Classification Numbers: 021, 022



1. Introduction.

Most of the existing literature on the stability of general equilibrium suffers from a common problem - the assumption that individual agents are unaware of the fact of disequilibrium.¹ Under tatonnement, agents take current prices as given and report their demands as though the economy were in Walrasian equilibrium. In "non-tatonnement", no-recontracting models (such as the Edgeworth process and the Hahn process), agents formulate demands again taking prices as given and paying no attention to the fact that they will often not be able to complete their planned transactions. In both types of models, the agents act as though they were in Walrasian equilibrium and simply fail to notice either that prices are not Walrasian and may change or that transactions may not be completed.²

Plainly, it is desirable to allow agents to have some idea of what is happening in disequilibrium, and this paper attempts to do so in one particular way,³ by allowing them to recognize that notional demands may not always be satisfied.⁴ Indeed, in one sense, we go to the other extreme, permitting agents fully to understand the mechanism through which expressed demands are translated into actual trades.

We consider a wide class of deterministic trading mechanisms for a pure-exchange economy. Each such mechanism takes the demands expressed by agents and produces actual trades which clear all markets. We assume the trading mechanism is common knowledge. Hence, agents take the mechanism into account when formulating their expressed demands.

Indeed, agents do more than that, for we assume that they understand not only how the trading mechanism generally works but also what its outcome will be in each instance. Therefore, given the prices, the expressed demands of all agents when transformed by the trading mechanism clear all markets, and agents get what they expect. Considering each trading moment as a game in which agents choose expressed demands as strategies and the market fine-tunes the trading mechanism as its strategy, agents and the market reach a Nash equilibrium at each moment of time. Nevertheless, there is "disequilibrium" in the sense that the trading mechanism distorts the opportunity sets from the usual price-taking budget sets, and agents end up with trades different from their notional demands.

The agents are required to have a large amount of information. In effect, they have momentary rational expectations of trade outcomes in this deterministic model. Note however, that while we allow the agents to have full information about current transaction difficulties, we leave them naive about the future. Specifically, agents think nothing will change in the future, so the current period may as well (from their viewpoint) be the last. Agents do not expect prices to change and hence do not speculate in this model (unlike that of Fisher, 1983).

Prices do change, nonetheless, reacting to signals given by the trading mechanism (the length of queues, for example). Hence, even though agents complete the transactions they expect, the economy does not stop moving until the trading mechanism ceases to produce disequilibrium signals, and notional demands, expresses demands and

actual trades all coincide. This feature of our model rules out non-Walrasian rest points.

Our principal result is that such a model is an Edgeworth process and hence globally stable. Naturally, given the way we have set it up, the equilibrium to which the model converges is Walrasian.⁵

2. The Deterministic Disequilibrium Awareness Model.

The central feature of our model is the specification of a wide class of deterministic trading mechanism. We assume pure exchange. Let z^i denote the vector of expressed demands (net of current stock) by the i^{th} agent. Then a trading mechanism is a pair of functions (h, c) . The function $h: \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is the final trade function, which for an arbitrary parameter $\tau \in \mathbb{R}^n$ takes expressed demand z^i and assigns final trade $y^i = h(\tau, z^i)$. Typically, the trading function might reduce the absolute value of the expressed demands. Given a price vector p , the function $c: \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ gives the additional deadweight cost imposed by the trading mechanism, so the total net costs of final trade y^i is

$$C(\tau, y^i; p) \equiv p \cdot y^i + c(\tau, y^i; p) . \quad (1)$$

Typically, purchases will have a "marginal cost" $\partial C / \partial y_j \geq p_j$, while sales will have a "marginal revenue" $\partial C / \partial y_j \leq p_j$, where by marginal revenue is mean the marginal reduction in net costs. We require the functions $h()$ and $c()$ to be Lipschitzian continuous (hence

differentiable almost everywhere). When applicable, we assume the deadweight costs are paid in a numeraire commodity (say n).⁶

Given a price vector p and initial stock ω^i , each agent is assumed to choose an expressed demand z^i that maximizes a twice-differentiable, strictly increasing,⁷ strictly quasi-concave utility function $u^i(\omega^i + y^i)$, subject to two constraints: (1) $C(\tau, y^i; p) \leq 0$, and (2) $y^i = h(\tau, z^i) \geq -\omega^i$, where the last inequality corresponds to a non-negative consumption set.

An important concept is the "virtual price" of trade. Our assumption that $c(\tau, \cdot; p)$ is Lipschitzian means that left and right hand derivatives always exist. We define virtual price as $\partial C / \partial y$ plus the shadow price of the quantity constraint $[y^i = h(\tau, z^i) \geq -\omega^i]$. Let $\beta_j^i(\tau, p, y^i)$ denote the virtual price of commodity j obtained by approaching y_j^i from the positive direction; and let $\gamma_j^i(\tau, p, y^i)$ denote the virtual price of commodity j obtained by approaching y_j^i from the negative direction. The Kuhn-Tucker conditions require $\lambda^i \gamma_j^i(\tau, p, y^i) \leq \partial u^i / \partial \omega_j^i \leq \lambda^i \beta_j^i(\tau, p, y^i)$, where λ^i is the Lagrangian multiplier associated with i 's budget constraint.

We shall assume orderly markets: that only one side of the market is "rationed". A buyer is rationed in commodity j if $y_j^i > 0$ and $\beta_j^i > p_j$. A seller is rationed in commodity j if $y_j^i < 0$ and $\gamma_j^i < p_j$.⁸ If some buyer (seller) is rationed in some commodity, then no sellers (buyers) are rationed in that commodity.

We adopt the convention that $\tau_j > (<) 0$ whenever buyers (sellers) of commodity j are rationed. Consequently, $\tau_j < 0$ implies $\gamma_j^1 < p_j = \beta_j^1$; $\tau_j > 0$ implies $\gamma_j^1 \leq p_j < \beta_j^1$; and $\tau_j = 0$ implies $\gamma_j^1 \leq p_j = \beta_j^1$. Further, when $\tau_j \geq 0$ and i has positive stock of j ($\omega_j^1 > 0$), then $\gamma_j^1 = p_j$.

Let $B(\tau, p)$ denote the opportunity set: the set of feasible trades (y^1) for agent i which satisfy the budget and quantity constraints imposed by the trading mechanism. We assume that $B(\cdot, \cdot)$ is non-empty, compact-valued, convex-valued, and continuous for all $p \gg 0$.⁹

Under these conditions, by standard proof, there is a unique continuous "target trade" function $\phi^i(\tau, p)$ which gives the trade y^1 that maximizes $u^i(\omega^1 + y^1)$ subject to the constraints embodied in $B(\tau, p)$. Moreover, any expressed demand z^1 such that $h(\tau, z^1) = \phi^i(\tau, p)$ is optimal. Let $\zeta^i(\tau, p) \equiv \{z^1 \mid h(\tau, z^1) = \phi^i(\tau, p)\}$ denote the expressed demand correspondence. When $\tau = 0$, the opportunity set is just the usual price-taking budget set, so $\phi^i(0, p) = \zeta^i(0, p)$ is the notional demand.

We now aggregate trade behavior. Define

$$\begin{aligned}\phi(\tau, p) &\equiv \sum_{i=1}^N \phi^i(\tau, p) \quad , \quad \text{and} \\ \zeta(\tau, p) &\equiv \sum_{i=1}^N \zeta^i(\tau, p) \quad .\end{aligned}\tag{2}$$

Note that when $\tau = 0$, we have $\phi(0, p) = \zeta(0, p)$, the aggregate notional demand. If, in addition, for some p^* , $0 = \phi(0, p^*) = \zeta(0, p^*)$,

then p^* is by definition a Walrasian equilibrium price. Let W denote the set of Walrasian prices.

For those trading mechanisms which impose deadweight costs when $\tau \neq 0$ (direct costs as in queue-rationing in contrast to simply utility losses), there will be a positive aggregate deadweight cost:

$$c(\tau, p) \equiv \sum_i c[\tau, \phi^i(\tau, p); p] . \quad (3)$$

In such cases, it is too much to ask that final trades $\phi(\tau, p) = 0$, since this condition is incompatible with Walras' Law and the payment of the deadweight costs: $p \cdot \phi(\tau, p) + c(\tau, p) = 0$. The most we can require is that $\phi_j(\tau, p) = 0$ for all $j \neq n$ (where n is the numeraire in which deadweight costs are paid), implying via Walras' Law that $p_n \cdot \phi_n(\tau, p) + c(\tau, p) = 0$. That is, $\phi(\tau, p) = (0^{n-1}, -c(\tau, p)/p_n)$, where 0^{n-1} is the zero vector in \mathbb{R}^{n-1} .

More generally, we say that (h, c) is an effective trading mechanism if for every price $p \gg 0$, there exists a $\tau^*(p)$ such that $\phi[\tau^*(p), p] + (0^{n-1}, c[\tau^*(p), p]/p_n) = 0$. In other words, for any arbitrary positive price there is a parameter τ^* such that final trades "clear" (subject to deadweight costs). These are really the only interesting trading mechanisms. If final trades do not clear, the trading mechanism is incompletely defined. Should expressed demand exceed supply in some commodity, what is the final outcome? The resolution of such discrepancies by definition yields a deterministic vector y^i for each agent such that the aggregate final trades "clear"; hence, the total process would be an effective trading mechanism.

Further, we say that (h, c) is a regular effective trading mechanism if (a) $\tau^*(p)$ is a Lipschitzian continuous function, (b) $\tau^*(p) = 0$ iff $p \in W$, and (c) $\tau_n^*(p) = 0$ independent of p . Condition (c) says that the numeraire commodity is unrationed, which is a typical result in the rationing literature. Regularity is a technical condition that ensures "nice" behavior and should hold for a large class of "smooth" trading mechanisms.¹⁰

To summarize, agents know the trading mechanism, and take p and $\tau^* [= \tau^*(p)]$ as fixed. [Note that we have implicitly assumed that τ^* is perfectly observable by all consumers; otherwise, the optimization problem involving $h(\tau^*, z^i)$ would be incompletely specified.] Agents choose an optimal expressed demand, $z^i(\tau^*, p)$, while they expect to get $\phi^i(\tau^*, p) = h[\tau^*, z^i(\tau^*, p)]$ less the deadweight costs. Then the trading mechanism determines final trades such that all markets "clear": $\phi(\tau^*, p) = (0^{n-1}, -c(\tau^*, p)/p_n)$, so every agent get what he or she expects. Further, $\tau^*(p) = 0$ iff p is Walrasian, in which case notional demands, expressed demands and final trades all coincide.

It is important to recognize that whether or not actual final trades $\phi^i(\tau^*, p)$ are the same as expressed demands $z^i(\tau^*, p)$, all agents correctly foresee their final trades, and hence satisfy their budget constraints and realize their anticipated utility $u^i(\omega^i + y^i)$. Consequently, utility is not declining out of Walrasian equilibrium, and since this is the crucial step in showing that the Hahn process is globally stable, the reader may begin to suspect that global stability

does not hold. On the other hand, trade is voluntary so in a sense utility must be increasing which is a crucial step in showing that the Edgeworth process is globally stable.

3. Dynamics.

In general, we may want to use information about both the trading mechanism's signal (τ^*) and aggregate expressed demand (ζ). However, since $\tau^*(p) = 0$ iff p is Walrasian, it will suffice for our purposes to assume prices respond to τ^* only. To this end, let $dp/dt = f(\tau)$, where $f()$ is continuous and $f(\tau) = 0$ iff $\tau = 0$. Then, the set of stationary points are exactly W . Given orderly markets and our sign convention on τ , it is natural to assume that $f()$ is sign preserving in the sense that $f_j(\tau)$ has the same sign as τ_j , which we henceforth do. [A special case would be to have $f_j()$ depend only on τ_j and to preserve sign.] Furthermore, we assume a minimum speed of adjustment ($\sigma > 0$) such that $|f_j(\tau)| \geq \sigma |\tau_j|$.

In the static analysis of section 2, the endowments of agents were suppressed from the notation. We must now incorporate them into the notation. Let $\underline{\omega} \equiv \{\omega^1, \dots, \omega^N\}$ denote the distribution of stock holdings. For every $\underline{\omega}$, the static solution in more explicit notation is $\tau^*(p, \underline{\omega})$. Regularity now entails that $\tau^*(\cdot, \cdot)$ is jointly Lipschitzian.

With trade in stocks, we must consider the dynamic path of $\underline{\omega}$ as

well. Interpreting y^i as instantaneous trade, $d\omega^i/dt = y^i = \phi^i(\tau, p, \omega^i)$. Define $G(p, \underline{\omega}) \equiv (\phi^1[\tau^*(p, \underline{\omega}), p, \omega^1], \dots, \phi^N[\tau^*(p, \underline{\omega}), p, \omega^N])$ to be the vector of instantaneous trades expressed as a function of $(p, \underline{\omega})$ alone.

Next define $F(p, \underline{\omega}) \equiv f[\tau^*(p, \underline{\omega})]$. Our general dynamic adjustment process is then

$$\begin{aligned} dp/dt &= F(p, \underline{\omega}) \\ d\underline{\omega}/dt &= G(p, \underline{\omega}) \end{aligned} \quad (4)$$

Since $\tau^*(p, \underline{\omega})$ is jointly Lipschitzian, $F()$ and $G()$ are Lipschitzian, so there will exist a unique solution path $p(t, p_0, \underline{\omega}_0)$ and $\underline{\omega}(t, p_0, \underline{\omega}_0)$ continuous in $(p_0, \underline{\omega}_0)$ such that (i) $\partial p(t, p_0, \underline{\omega}_0)/\partial t = F[p(t, p_0, \underline{\omega}_0), \underline{\omega}(t, p_0, \underline{\omega}_0)]$ and $\partial \underline{\omega}(t, p_0, \underline{\omega}_0)/\partial t = G[p(t, p_0, \underline{\omega}_0), \underline{\omega}(t, p_0, \underline{\omega}_0)]$ for all $t > 0$, and (ii) $p(0, p_0, \underline{\omega}_0) = p_0$ and $\underline{\omega}(0, p_0, \underline{\omega}_0) = \underline{\omega}_0$.

The central question is whether such a process is globally stable; i.e. starting at arbitrary $(p_0, \underline{\omega}_0)$ does the process always converge to some Walrasian point?

We assume agents myopically maximize $u^i(\omega^i + y^i)$ with respect to y^i subject to $B(\tau, p)$. Without loss of generality, we choose an ordinal utility function that is bounded above to represent an agent's preferences.

Given strictly increasing utility defined on the non-negative orthant, it is reasonable to assume that agents always hold positive stocks of every commodity, so the short constraint $(y^i \geq -\omega^i)$ is never binding. We henceforth assume that $\underline{\omega}(t)$ is bounded away from zero in

every component for all $t > 0$. Then, given twice-differentiable utility functions, along this strictly positive path of $\underline{\omega}(t)$, the first and second derivatives of $u^i()$ are bounded.

Theorem 1. In our model of deterministic disequilibrium

awareness and trade in stocks, given a regular effective trading mechanism that satisfies orderly markets, and given the preceding assumptions, the dynamic system (4) is globally stable.

PROOF: At each stage, an agent is fully aware of the trading mechanism and makes a trade offer which ultimately changes his stock portfolio if and only if the correctly foreseen change will increase his utility. Therefore, u^i is always non-decreasing and is strictly increasing whenever $y^i \neq 0$. Thus, the temporal sequence of u^i values must converge monotonically to a limit, say \bar{u}^i . Moreover, given, strictly increasing utility, the corresponding sequence of y^i must converge to 0.

Recall the definitions of the virtual prices β_j^i and γ_j^i . For the natural numeraire (commodity n), $\tau_n = 0$ and $\omega_n^i > 0$; hence, $\gamma_n^i(\tau, p, y^i) = \beta_n^i(\tau, p, y^i) = p_n$ for all i and all (τ, p, y^i) . Thus, $\partial u^i / \partial \omega_n^i = \lambda^i p_n$. Now let $\mu_j^i \equiv (\partial u^i / \partial \omega_j^i) / (\partial u^i / \partial \omega_n^i)$ for $j \neq n$ denote the marginal rate of substitution evaluated at ω^i , and let $\hat{\gamma}_j^i \equiv \gamma_j^i / p_n$ and $\hat{\beta}_j^i \equiv \beta_j^i / p_n$. Then the Kuhn-Tucker conditions require $\hat{\gamma}_j^i \leq \mu_j^i \leq \hat{\beta}_j^i$ for all $j \neq n$ and every agent. It is convenient to let $\hat{p} \equiv p / p_n$ denote the relative price with respect to the numeraire.

Define $\alpha \equiv \max_{i,j} \{|\mu_j^i - \hat{p}_j|\}$. Note that $\alpha \geq 0$ and equal to zero iff $\tau = 0$. To see this, (1) if $\alpha = 0$, so $\mu_j^i = \hat{p}_j$ for all i and j , then

(by strict quasi-concavity of utility functions) the desired final trade is $y^1 = 0$ which is the same as the Walrasian notional demand [i.e. the current ω^1 is optimal with respect to $p \cdot y^1 \leq 0$], so p is a Walrasian price. But then by regularity of the trading mechanism, $\tau = 0$. (2) If $\tau = 0$, then $\hat{y}_j^1 = \mu_j^1 = \hat{\beta}_j^1 = \hat{p}$ for all i and j , so clearly $\alpha = 0$.

Further, $\alpha(t)$ is bounded. To see this, let r be a commodity that satisfies the definition of α . There are two cases in which α can be positive. (1) $\alpha = (\mu_r^1 - \hat{p}_r) > 0$ for some i , implying that $\tau_r > 0$. [If $\tau_r \leq 0$, then $\hat{y}_r^1 \leq \mu_r^1 \leq \hat{p}_r$.] But since $f()$ is sign preserving, $d\hat{p}_r/dt > 0$ [recall that since $f_n()$ is sign-preserving, $dp_n/dt = 0$]. Since $u^i()$ has bounded first and second derivatives along the path, the μ_j^1 are bounded, and hence \hat{p}_r is increasing, α is bounded above for the first case. (2) $\alpha = (\hat{p}_r - \mu_r^1) > 0$ for some i , implying that $\tau_r < 0$. [Recall that given $\omega_r^i(t) > 0$, $\hat{y}_r^1 < \hat{p}_r$ iff $\tau_r < 0$, and $\hat{\beta}_r^1 > \hat{p}_r$ iff $\tau_r > 0$.] But then $d\hat{p}_r/dt < 0$, so again α is bounded above.

Since $\alpha(t)$ is bounded, it has a non-empty set of limit points: the set of all accumulation points of $\alpha(t_\lambda)$, where $\{t_\lambda\}$ is a sequence of times such that $t_\lambda \rightarrow +\infty$ as $\lambda \rightarrow +\infty$. Let $\bar{\alpha}$ be a limit point, and suppose $\bar{\alpha} > 0$. Again let r be a commodity that satisfies the definition of α . Suppose $\bar{\alpha} = (\mu_r^1 - \hat{p}_r) > 0$ for some i , so $\tau_r \geq \varepsilon > 0$ [the ε depending on $\bar{\alpha}$]. Since $f()$ is sign preserving, $d\hat{p}_r/dt > 0$. Since $u^i()$ has bounded first and second derivatives, $d\mu_r^1/dt$ converges to zero. It follows (given the minimum speed of price adjustment) that there is a t' such that for all $t > t'$, $d\alpha/dt < -\sigma\varepsilon/2 < 0$. Similarly, if $\bar{\alpha} = (\hat{p}_r - \mu_r^1) > 0$ for some i , so $\tau_r \leq -\varepsilon < 0$, then $dp_r/dt < 0$; hence, again $d\alpha/dt < -\sigma\varepsilon/2 < 0$ for all $t > t'$. But $d\alpha/dt$ bounded below 0 is incompatible with $\bar{\alpha} > 0$ being a limit point. Therefore, $\bar{\alpha} = 0$ must be the only limit

point.

Since the μ_j^i are bounded and $\alpha(t)$ is bounded, clearly prices are bounded, so prices have a well-defined limit set. Moreover, since the μ_j^i are strictly positive, and $\alpha \rightarrow 0$, the limit prices are strictly positive. Since we have a pure exchange economy with fixed finite stocks, $\omega(t)$ is bounded and so has a well-defined limit set. Let (p', ω') and (p'', ω'') denote two limit points, where (p', p'') are derived from a subsequence of the respective sequences of t_λ which generated (ω', ω'') .

Now we have $u^i(\omega^{i'}) = u^i(\omega^{i''}) = \bar{u}^i$; i.e. the final stock allocations are on the same indifference surfaces. Moreover, since $\tau = 0$, total net costs $C(0, y^i; p') = p' \cdot y^i$. Letting $y^i = (\omega^{i'} - \omega^{i''})$, then by strict quasi-concavity, $p'(\omega^{i'} - \omega^{i''}) \leq 0$ with strict inequality unless $(\omega^{i'} - \omega^{i''}) = 0$ [recall that prices are strictly positive]. But $(\omega^{i'} - \omega^{i''})$ summed over all i is identically zero (by virtue of pure exchange), so $(\omega^{i'} - \omega^{i''}) = 0$ for all i . In other words, the stocks converge for each agent to (say) $\bar{\omega}^i$.

Given $\bar{\omega}^i \gg 0$ and strict quasi-concavity, the μ_j^i must also converge to a unique limit, which implies (since $\alpha \rightarrow 0$) that the prices must converge to a unique Walrasian equilibrium price. Q.E.D.

4. Conclusion.

We have approached the issue of disequilibrium awareness by specifying a general deterministic trading mechanism and supposing that all agents know the trading mechanism perfectly (albeit myopically). We defined a regular effective trading mechanism such that final trades clear for every price and assumed it has the "orderly markets" property. The observable parameter of the trading mechanism (τ) was a natural signal for price adjustment.

When trade is in stocks,¹¹ we found that the natural sign-preserving price adjustment process is globally stable. This result followed from a regularity condition on the trading mechanism and very mild conditions on agent preferences.

This global stability result for trade in stocks is an improvement over the received non-tatonnement results because agents make offers with perfect awareness of the trading mechanism, rather than blindly pursuing trade demands in complete ignorance of prices being non-Walrasian. Thus, the Edgeworth process is compatible with perfect disequilibrium awareness of this type.

On the other hand, this very result immediately implies that the Hahn process is not compatible with the kind of disequilibrium awareness studied here. This is because, with perfect awareness of the outcome of the trading mechanism, every agent gets what he or she expects. Nevertheless, the system keeps moving out of equilibrium,¹² but now

there is no reason for every agent to find that prices move perversely, as in the Hahn process. Indeed, there are "favorable surprises" (Fisher, 1983), although, of course, such surprises disappear asymptotically.

Evidently, disequilibrium dynamics are quite sensitive to the way in which agents understand what is going on. We have shown here that, in a wide class of models, allowing agents fully to understand the way in which expressed demands result in final trades leads to stability. We have not allowed agents to foresee the motion of the system, however; indeed, we have kept them from realizing that prices change, even though prices react to the very trading mechanism that agents are supposed to understand so well.

Our agents are thus unrealistically well-informed and sophisticated in some respects and unrealistically ignorant and naive in others. Further work in this area is highly desirable if we are ever to understand how (or if) real economies succeed in reaching equilibrium.

ENDNOTES

1. For surveys, see Arrow and Hahn (1971), Hahn (1982), and Fisher (1983).
2. There are two ways to interpret goods in non-tatonnement models. One is that the goods are perfectly durable commodities, but nobody eats before the process terminates so what matters is the total accumulated stock. The other interpretation is that the goods are "commodity consols" that promise delivery of a constant flow of perishable commodities or services. In both interpretations trade is in "stocks".
3. Fisher (1981, and especially 1983) gives a more ambitious, but not wholly satisfactory attempt to deal with the problem of disequilibrium awareness, permitting agents to expect price change as well as transaction restrictions.
4. "Notional demand" is the traditional price-taking demand with no disequilibrium awareness. In contrast, "expressed demand" is the demand actually expressed (and acted upon) by the agent given his awareness of what happens in disequilibrium (such as rationing). The agent may expect to get something different from the demand he expresses; this expectation is referred to as "expected trade" or "target trade". Following the expression of demands, we get "actual trades".
5. Whether that result would continue to hold if agents were permitted to expect price changes is at best doubtful. The Edgeworth process is not directly suited to situations of speculation and arbitrage. See Fisher (1983, pp 30-31).

6. An example of such a deterministic trading mechanism is queue-rationing [Stahl (1985)]. To transform the queue-rationing model into this framework, interpret the expressed demand for time (z_n) to be the demand for time excluding the requirements of the queues. Then the only expressed demands that are altered by the trading mechanism are those for time; final trade in time is $z_n - Q(\tau, z)$, where Q is total queue time for trade z . The total net cost function is $p \cdot y + Q(\tau, y)$. A coupon-rationing mechanism (e.g. Hahn, 1978) with tradeable coupons of intrinsic value (as cigarettes in post-WWII Germany) is formally equivalent to queue-rationing. A third example is the quantity-rationing mechanism of Drèze (1975) for which $c(\tau, y^i; p) = 0$. The quantity limit on purchases can be defined as $B_j \equiv L - \max\{0, \tau_j\}$, and for purchases $S_j \equiv L - \max\{0, -\tau_j\}$, for some appropriately large positive constant L .
7. Strictly increasing utility is commonly assumed [Uzawa, 1960; Arrow and Hahn, 1971], and rules out free goods. We suspect a weaker assumption would suffice but not without tedious technical complications.
8. Note that if an agent wants to sell j but has no stock, then the virtual price is less than p_j , but this is not due to the trading mechanism, so it is not considered "rationing".
9. This is an implicit restriction on the trading mechanism. All the mechanisms mentioned in footnote 6 have this property.
10. For example, the aforementioned queue-rationing and coupon-rationing trading mechanisms are regular and effective for non-critical economies. For Drèze quantity-rationing, Lipschitzian continuity holds everywhere except on a set of measure zero that is

inconsequential to the dynamics. Lipschitzian continuity is imposed for convenience only. Since $\tau^*(p)$ is always upper hemi-continuous, we could employ the extended Liapounov techniques of Champsaur, Drèze and Henry (1977). Note, however, that a coupon-rationing mechanism with marketable coupons at a fixed positive price is not a regular trading mechanism because if $p \in W$ and $\tau = 0$ there will be excess expressed demand since everyone will want to sell coupons and purchase more goods. On the other hand, if the coupon's market value is endogenized (as in a black market) and if τ is interpreted as the "money" value of coupon surcharges, then regularity is restored. On the other hand, "deterministic proportional rationing" (which gives every agent a fraction of his expressed demand) is not an effective trading mechanism because for every $p \in W$ and τ , the mechanism can be completely undone: there is an expressed demand z^i such that $h(\tau, z^i) = \phi^i(0, p)$ the notional demand [see Benassy (1977)].

11. Observing that a significant portion of trade in the real world is in spot flows and not stocks (or commodity consols), it would be desirable to study economies in which trading is in flows. In such economies, the income effects that plague traditional tatonnement theory continue to be troublesome in the presense of disequilibrium awareness. While conditions for local stability (e.g. eigenvalue conditions) can be easily stated, the results are far from a satisfactory answer to stability questions.
12. Fisher (1983, pp. 181-184) points out that perfect foresight about the ability to transact in disequilibrium (here, perfect awareness of the trading mechanism) is likely to be uninteresting as it

implies that the system never moves. That result, however, applies where the system (prices, in particular) reacts to the difference between agents' expected (target) trade and actual trade, and this difference is identically zero given perfect awareness. It is avoided here because prices react to disequilibrium signals (namely τ^*) stemming from the difference between actual trades, $\phi^i(\tau^*, p)$, and notional demands, $\phi^i(0, p)$, and this difference vanishes if and only if $\tau^* = 0$. Hence, prices keep moving even though markets clear in the sense that the trading mechanism always gives agents what they expect.

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